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Some Remarks on the Neutrino Oscillation Phase in a Gravitational Field

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Abstract

The weak gravitational field expansion method to account for the gravitationally induced neutrino oscillation effect is critically examined. It is shown that the splitting of the neutrino phase into a "kinematic" and a "gravitational" phase is not always possible because the relativistic factor modifies the particle interference phase splitting condition in a gravitational field.

Key words: neutrino oscillation, interference phase, weak field.

The gravitationally induced neutrino oscillation phase has attracted much attention in recent years [1–11]. However, a lot of problems on the weak field expansion method still exists. In order to clarify some of the conceptual problems appearing on the interplay of gravity and neutrino oscillation, we critically examine in this paper the weak-field expansion method used to take into account the gravitational effect on the neutrino oscillation phase. We set $G = \hbar = c = 1$ throughout the manuscript.

For one massive neutrino produced at the source position A, with the detector at position B, the geometrical optics phase in a curved spacetime can be calculated by the conventional formula [4,8,12–14]

$$\Phi = \int_A^B m ds = \int_A^B g_{\mu\nu} P^\mu dx^\nu \ , \tag{1}$$

where P^ν is the 4-momentum, $g_{\mu\nu}$ is the metric and ds is the spacetime line element. For the case of two massive neutrinos interference problem, however, a covariant neutrino wave-packet approach should be introduced to calculate Φ.

To study the thermal neutron interference of COW experiment [15] in the weak field limit [12], the phase factor (1) can still be used. In the Earth's gravitational field, the neutron interference phenomenon is usually calculated by inserting a Newtonian potential in the Schroedinger equation [16]. This, however, precludes the description of such interference effect when the gravitational field is not Newtonian. Another approach to calculate the thermal neutron interference phase, which takes into account the full tensor character of gravitation, can alternatively be used [12]. Following this approach, we consider the linearized gravitational field (weak field), and call $h_{\mu\nu}$ the small deviation from the Minkowski metric $\eta_{\mu\nu}$, so that $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu} = -\alpha\delta_{\mu\nu}$, with $\alpha = r_s/r$ and $r_s = 2M$ the Schwarzschild radius of the gravitational source mass M. When dealing with the thermal neutron interference, the phase of Eq.(1) may be split up into a "kinematic" phase Φ^0 and an extra "gravitationally induced" phase Φ^G [12]:

$$\Phi = \Phi^o + \Phi^G \ . \tag{2}$$

Writing

$$ds^2 = (ds^o)^2 + h_{\mu\nu} dx^\mu dx^\nu \ , \tag{3}$$

where $(ds^o)^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ is the flat spacetime interval, the weak field induced interference phase splitting expansion reads

$$ds = [(ds^o)^2 + h_{\mu\nu} dx^\mu dx^\nu]^{1/2} \approx ds^o + \frac{1}{2} h_{\mu\nu} \frac{dx^\mu}{ds^o} dx^\nu \ . \tag{4}$$

We then find that $\Phi^o = \int_A^B m ds^o$ is the phase in the flat spacetime, and

$$\Phi^G = \frac{1}{2} \int_A^B h_{\mu\nu} P^\mu_{(o)} dx^\nu \ , \tag{5}$$

where $P^\mu_{(o)} = m dx^\mu / ds^o$, is the usual 4-momentum of special relativity. For the case of neutrons with not too large translational velocities ($v^2 \sim 10^{-10}$) [12,15], the above treatment is completely equivalent to that using the Newtonian potential, and accounts correctly for the thermal neutron interference experiment [12].

The weak field approximation is a powerful tool to work out gravitationally related problems [17], especially those related to neutron optics [12,13,15]. On the other hand, when dealing with the neutrino phase factor, the weak field approximation to account for the interference phase splitting, given by Eqs.(3) and (4), should be carefully considered. Before getting to the point, it is helpful to keep in mind that, unlike the thermal neutron, massive neutrinos propagate at nearly the speed of light, being a relativistic object with much higher momentum-energy than its rest mass-energy.

The weak field induced phase splitting originates from the fact that the first term in the r.h.s. of Eq.(3) is much greater than the second one. This approximation is satisfactorily applied to the case of the neutron optics because thermal neutrons are low-energy objects. But, for an extremely relativistic object, the weak field induced phase splitting requires further considerations. In order to compare the magnitudes of the two terms in the r.h.s. of Eq.(3), we take the ratio between them

$$\xi = \frac{|h_{\mu\nu} dx^\mu dx^\nu|}{(ds^o)^2} = \frac{\alpha \delta_{\mu\nu} dx^\mu dx^\nu}{(ds^o)^2} = \frac{\alpha \delta_{\mu\nu} dx^\mu dx^\nu}{ds^2} \frac{ds^2}{(ds^o)^2} . \tag{6}$$

We can then write

$$\frac{ds}{ds^o} = \sqrt{1-\xi}. \tag{7}$$

and consequently, we obtain

$$\xi = \frac{\alpha}{m^2}(1-\xi)\,[(P^o)^2 + (P^r)^2], \tag{8}$$

where the momentum is defined as $P^\mu = m(dx^\mu/ds)$, and $(P^r)^2 = (P^x)^2 + (P^y)^2 + (P^z)^2$. We then get

$$\xi = \frac{\alpha(2\gamma^2-1)}{1+\alpha(2\gamma^2-1)}\,, \tag{9}$$

where $\gamma = (P^o/m)$ is the relativistic factor. The approximated mass shell condition $(P^o)^2 - (P^r)^2 \approx m^2$ has been used in the above expression.

From Eq.(7) we can see that the value of ξ must be in the range $0 \le \xi \le 1$, where the values 0 and 1 represent respectively the vacuum case and the null case (light trajectory). In the absence of gravitational field, which corresponds to $\alpha \to 0$, we find $\xi = 0$, and the vacuum situation is recovered. When $v \to c$, which corresponds to an ultra high-energy case, $\gamma \to \infty$, and we obtain $\xi = 1$. So, we see from Eq.(9) that in fact $0 \le \xi \le 1$ in any situation.

We can now discuss the expansion conditions. For a low energy object, as for example a thermal neutron in the laboratory, whose typical velocity is $v^2 \sim 10^{-10}$, we have $\gamma \simeq 1$, and consequently

$$\xi = \frac{\alpha}{1+\alpha} \simeq \frac{r_s}{r} \ll 1. \tag{10}$$

This is the conventional weak field condition, which means that the phase splitting can be performed. On the other hand, for high-energy massive neutrinos, as for example an electron neutrino, the relativistic factor is $\gamma^2 \sim 10^{12}$ [18,19], and in the case of the Earth gravitational potential, for which $(r_s/r) \sim 10^{-11}$, we get $\xi = 0.95 \approx 1$. Consequently, the phase splitting of Eq.(4) or Eq.(7) can not be performed. For a galaxy, the sun, a white dwarf and a neutron star, the gravitational potentials are respectively 10^{-7}, 10^{-6}, 10^{-3} and 10^{-1}. In all these cases, the electron neutrino does not admit the conventional phase splitting as the thermal neutron does. We can then conclude that the particle interference phase splitting depends on the relativistic factor, and that any interference phase splitting for high energy particles which does not take into consideration the relativistic factor, will be meaningless.

Neutrinos are extremely relativistic particles with very large relativistic factors. Consequently, for these particles, the interference phase splitting will not work for most of the usual astrophysical situations. In other words, the conventional interference phase splitting condition applied to the thermal neutron interference experiment can not be directly applied to a neutrino because the relativistic factor modifies the interference phase splitting condition. As an immediate consequence, the splitting of the neutrino phase into "kinematic" and "gravitational" phases is not possible for both the neutron star and the Earth gravitational fields, as is sometimes claimed in the literature [1–3]. On the other hand, thermal neutrons in COW experiment are low energy objects whose relativistic factors are nearly unity, and the gravity weak-field limit yields a critical gravitational potential of order $(r_s/r)_{critical} \sim 1$. For these particles, therefore, one does not need to consider the relativistic factor in the interference phase splitting.

As a further remark, we note that the phase factors Φ of Eq.(1) is not the total phase of the neutrino in a gravitational field, but only the phase due to the coupling of the energy-momentum of the particle to the (curved) spacetime geometry, and named type-I phase [13]. Concerning the spinning aspect of the particle, gravitation gives rise also to a type-II phase, a phase shift related to the coupling of the "spin connection" to the curvature. The general physical characters of these two types of phase factors can be found in Ref. [13]. It should be mentioned, however, that the type-II phase has no contribution to the neutrino oscillation phase in a Schwarzschild spacetime because of the static and spherical symmetry [4].

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